2.5 Bayesian reasoning...

We are given initially

$$\text{prob}(+|\text{disease}) = 0.9$$

Bayes:

$$\text{prob}(+|\text{disease})\text{prob}(\text{disease}) = \text{prob}(\text{disease}|+)\text{prob}(+)$$

or

$$\text{prob}(\text{disease}|+) = \frac{\text{prob}(+|\text{disease})\text{prob}(\text{disease})}{\text{prob}(+)}$$

Find the denominator from normalization

$$\text{prob}(\text{disease}|+) + \text{prob}(\text{no disease}|+) = 1$$

giving the final result

$$\text{prob}(\text{disease}|+) = \frac{\text{prob}(+|\text{disease})}{\text{prob}(+|\text{disease}) + \text{prob}(+|\text{no disease})\left(\frac{\text{prob(no disease)}}{\text{prob(disease)}}\right)}$$

depending on the false-alarm rate and the general incidence of the problem.
e.g. if prob(disease)=0.1 and the false-alarm rate prob(+|no disease) = 0.01

$$\text{prob}(\text{disease}|+) = 0.91.$$

On the other hand, if the false-alarm rate were 0.1, then

$$\text{prob}(\text{disease}|+) = 0.5.$$

Think of the second case this way. Take 1000 people; 100 of them (1000×0.1) have the disease. Of these, 90 or (100×0.9) give a positive test. However, another 90 people ((1000-100)×0.1) give a positive test without having the disease. So, given a positive test, a fraction 90/(90+90) actually have the disease.